

## ASTRO 501

### Catalogs, Map Projections, Standard Coordinates

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#### 1. The PPM catalog

The *PPM Star Catalogue* gives **P**ositions and **P**roper **M**otions of 181731 stars north of  $-2.5^\circ$  declination for the equinox and epoch J2000.0. A copy of this 4-volume work is available in Davey Lab, Room 530; the title page and first catalog page are reproduced here, to show the kinds of information that such catalogs contain.

Before you make use of *any* data source, whether printed or on-line, you should familiarize yourself with any introductory material provided. This will explain the meaning of the columns, give guidance as the use of the material, and provide warnings about when the material should *not* be used.

In the case of the *PPM Star Catalogue*, the introductory material will indicate the units of measurement for the various tabulated quantities. Note that many different names exist for the same star, and the more common catalog designations for stars are provided in *PPM* for cross-reference. Note how precession has changed the coordinates of the stars — Entry #1 = DM +82 0746 (in Decl. strip  $+82^\circ$  in 1855) now has Decl.  $+83^\circ$ , for example.

Note that *equinox J2000.0* refers to the coordinate system that is used, while *epoch J2000.0* refers to the date for which the mean positions of stars are tabulated. The actual observations of the star positions were accumulated over time, and then extrapolated to J2000.0. The columns *EPA* and *EPD* give the *mean epoch* of observations that were used to derive the R.A. and Decl. of each star for the tabulated epoch (leading digits “19” of year suppressed for compactness). Note how old some of these are! The HIPPARCOS satellite has provided updated positions and proper motions for most of these stars, on a more accurately defined coordinate frame (the ICRF).

# PPM STAR CATALOGUE

## Positions and Proper Motions

of 181731 stars north of  $-2.5$  degrees declination  
for equinox and epoch

J2000.0

compiled by  
Siegfried Röser and Ulrich Bastian

Volume I:

Zones  $+80^\circ$  to  $+30^\circ$

PPM J2000.0

+80 degrees

Stars No. 001 ff

PPM	DM	Mag Sp	R.A. J2000	Dec. J2000	PMA	PMD	N	SA	SD	SPHA	SPHD	EPA	EPD	SAD	HD	AGK3	Notes
1	+82 0746	11.3 K5	0 0 4.606	+83 6 19.09	0.0036	-0.022	4	11	12	4.9	5.4	22.20	24.00	4009		+82 0730	
2	+85 0410	10.5 G5	0 0 57.917	+86 40 20.03	0.0144	0.092	5	07	08	3.5	4.0	39.69	36.77	4010		+86 0328	
3	+84 0542	10.6 G5	0 1 27.925	+85 20 11.40	0.0176	0.031	5	07	08	3.6	4.0	39.75	36.81	4011		+85 0402	
4	+82 0747	11.0 F5	0 1 44.904	+83 13 37.63	0.0254	-0.006	4	11	12	4.8	5.2	23.32	25.09	4012		+82 0731	
5	+83 0672	11.3 K0	0 1 47.495	+83 52 52.61	0.0145	0.005	4	11	12	5.0	5.4	22.75	24.51	4013		+83 0616	
6		11.1 A2	0 2 5.001	+80 13 9.32	-0.0041	-0.006	5	11	12	4.8	5.3	23.43	25.64	10934		+79 0698	
7	+79 0798	11.3 G5	0 2 17.192	+80 41 48.33	-0.0012	-0.013	5	12	12	4.9	5.3	22.56	24.84	4014		+80 0508	
8	+84 0347	8.0 F0	0 2 27.759	+87 1 57.84	0.0167	0.011	6	07	08	3.3	3.6	39.45	36.18	4015	224991	+86 0329	
9	+82 0748	7.3 A0	0 2 42.606	+82 58 22.99	0.0031	-0.015	5	09	10	3.8	4.2	23.72	26.19	1	225019	+82 0001	
10	+79 0799p	8.1 F2	0 2 46.544	+80 16 55.61	0.0190	-0.001	8	05	07	2.1	2.3	53.65	51.35	2	225020	+80 0001	DM
11	+79 0799s	10.4	0 2 48.940	+80 17 11.49	0.0157	0.007	5	10	11	4.6	4.7	23.76	23.89			+80 0002	D
12	+81 0841	9.6 B8	0 3 16.517	+82 39 22.89	-0.0084	-0.024	4	10	10	4.4	4.6	27.87	27.00	3		+82 0002	D
13	+79 0800	10.2 F0	0 3 17.608	+80 29 6.99	0.0088	-0.007	5	11	11	4.8	5.0	27.08	27.44	4		+80 0003	
14	+79 0801	10.7 K0	0 3 29.648	+80 44 16.25	0.0031	0.008	6	09	09	2.6	2.8	44.51	45.32	5		+80 0004	H
15	+85 0411	10.4 F0	0 3 44.408	+86 22 20.87	-0.0147	-0.008	5	07	08	3.6	4.0	40.01	37.07	6		+86 0001	
16	+84 0543	11.0 K2	0 4 5.761	+85 14 57.09	0.0061	-0.008	4	11	12	4.9	5.4	24.13	25.81	7		+84 0001	
17	+83 0673	11.1 M1	0 4 20.931	+84 28 23.66	-0.0017	-0.010	5	11	12	4.9	5.3	23.42	25.16	8		+84 0002	
18	+82 0749	11.7 K0	0 4 42.108	+82 52 49.13	0.0033	-0.025	4	12	12	5.2	5.3	24.07	23.35	9		+82 0003	
19	+82 0750	10.7 F0	0 4 43.139	+83 39 26.07	-0.0122	0.002	4	11	11	4.8	5.0	25.04	25.20	10		+83 0001	
20	+81 0843	11.2 G0	0 5 39.852	+82 21 14.97	0.0130	0.020	4	11	12	5.0	5.5	23.51	25.40	12		+82 0005	
21	+87 0220	10.1 K0	0 6 10.994	+87 53 22.33	-0.0279	0.023	8	04	06	1.9	2.2	53.01	51.72	14		+87 0001	H
22	+84 0545	11.1 G5	0 6 32.164	+85 39 12.46	-0.0708	0.145	5	07	09	3.6	4.2	39.41	37.53	15		+85 0001	
23	+81 0846	10.6 A5	0 6 53.706	+82 41 53.02	-0.0072	-0.011	4	11	11	4.7	4.9	24.83	24.98	16		+82 0006	
24	+84 0546	8.7 A3	0 7 6.751	+85 24 17.87	0.0297	0.011	6	06	08	3.0	3.6	36.63	35.91	17	174	+85 0002	
25	+79 0803	10.6 G5	0 7 10.045	+80 43 46.56	-0.0058	-0.016	5	11	11	4.6	4.9	25.06	26.21	18		+80 0006	
26	+83 0674	12.1 K2	0 7 20.583	+84 25 11.09	-0.0024	-0.020	4	12	13	5.2	5.7	25.04	26.70	19		+84 0003	
27	+83 0675	12.2 K0	0 7 44.925	+84 36 15.86	-0.0011	-0.015	4	12	13	5.3	5.9	25.41	27.03	20		+84 0004	
28	+84 0547	11.1 K0	0 7 54.039	+85 45 44.37	-0.0257	0.007	6	06	08	2.6	2.9	47.55	47.84	21		+85 0003	H
29		10.9 F8	0 8 18.555	+80 10 1.37	-0.0072	-0.037	5	11	12	4.7	5.0	23.59	24.82	4041		+79 0002	
30	+85 0412	9.0 G0	0 8 50.152	+86 47 16.43	0.3811	-0.003	6	06	08	3.2	3.5	38.55	34.48	22	245	+86 0002	
31	+79 0001	10.0 A0	0 9 28.619	+80 24 44.80	-0.0046	-0.028	5	11	11	4.5	4.7	26.32	26.80	23		+80 0007	
32	+79 0002	10.3 K0	0 9 53.429	+80 25 54.01	0.0013	-0.019	5	11	11	4.5	4.8	25.45	26.13	25		+80 0008	
33	+85 0001	9.9 F8	0 10 4.441	+86 1 23.31	0.0135	0.006	5	07	08	3.5	3.9	39.90	37.00	24		+85 0004	
34	+83 0001	9.1 F5	0 10 5.004	+84 9 7.44	0.0187	0.007	4	10	10	4.6	4.6	29.54	28.37	26	486	+83 0002	
35	+81 0001	10.7 G5	0 10 28.002	+82 43 29.69	0.0261	-0.019	4	11	11	4.8	4.9	24.56	24.71	27		+82 0007	
36	+82 0001	10.4 F2	0 11 10.279	+83 3 48.74	-0.0042	-0.020	4	10	11	4.7	4.9	25.71	25.71	28		+82 0008	
37		10.3 A0	0 11 55.328	+80 17 37.11	0.0076	-0.005	5	11	11	4.5	4.8	25.66	26.14	30		+80 0009	
38	+81 0002	11.7 K0	0 11 55.610	+81 58 52.34	-0.0103	-0.022	4	12	12	5.2	5.4	24.41	24.59	29		+81 0001	
39	+82 0002	10.2 K2	0 12 24.523	+83 11 43.58	0.0186	0.003	5	10	10	4.2	4.4	22.23	22.23	32		+82 0009	D
40	+83 0002	11.1 K0	0 12 30.731	+84 31 32.64	0.0149	-0.003	4	11	12	4.9	5.3	23.41	25.15	31		+84 0005	
41	+84 0002	11.0 A0	0 12 55.049	+84 57 54.11	-0.0078	-0.015	4	11	12	4.9	5.4	24.13	25.81	33		+84 0006	
42	+81 0003	10.7 F0	0 12 59.756	+82 1 23.04	-0.0035	-0.011	4	11	11	4.9	5.1	25.42	25.90	35		+81 0002	
43	+83 0003	9.7 F5	0 13 8.824	+84 2 38.29	0.0761	-0.032	5	09	10	4.0	4.2	23.34	23.26	34		+83 0003	
44	+79 0003	10.2 G5	0 14 35.018	+80 28 55.04	-0.0042	-0.006	5	11	11	4.5	4.7	25.87	26.36	37		+80 0010	
45	+82 0003	10.7 K2	0 15 48.087	+83 23 27.49	-0.0101	-0.010	5	10	10	4.3	4.5	20.41	20.77	40		+83 0004	
46	+87 0001	9.3 A2	0 16 4.806	+88 24 4.46	0.0044	-0.002	9	05	07	2.4	2.9	32.81	27.86	36	815	+88 0001	
47	+82 0004	9.4 F0	0 16 5.891	+83 22 12.48	-0.0201	-0.019	5	09	10	4.1	4.1	25.33	23.94	42		+83 0005	
48	+85 0003	9.1 G0	0 14 7.792	+84 43 33.61	0.1016	-0.002	5	07	08	3.6	3.9	41.01	37.55	38	946	+86 0003	

## 2. Standard Equinoxes, Precession, Recent Catalogs & Surveys, and Galactic Coordinates

### Standard Equinoxes and Example Catalogs:

1855 – Bonner Durchmusterung (BD)

1875 – Cape, Córdoba Durchmusterungen (CD, CoD: southern sky)

1900 – Henry Draper catalog (HD), etc.

1950 – now B1950, or ‘B’ prefix [if no prefix, B is assumed.] (**B**esselian year 1950.0).

Example: PG 0900+400 (Palomar-Green, or PG catalog).

2000 – ‘J’ prefix [mandatory!] (**J**ulian year 2000.0).

Example: RX Jhhmmss.ss±ddmmss.s (ROSAT X-ray source).

### Examples of Precession:

0. Note the *size* and *sign* of the precessional change.

1. Rigel =  $\beta$  Orionis = HD 34085:

1900 :	05 <sup>h</sup>	09 <sup>m</sup>	43.8 <sup>s</sup>	−08°	19′	01″	(HD)
1950 :	05 <sup>h</sup>	12 <sup>m</sup>	07.99 <sup>s</sup>	−08°	15′	28.6″	(FK4)
2000 :	05 <sup>h</sup>	14 <sup>m</sup>	32.27 <sup>s</sup>	−08°	12′	05.91″	(ICRS)

2. PG 1224+309 (DA + dM binary):

1950 :	12 <sup>h</sup>	24 <sup>m</sup>	01.6 <sup>s</sup>	+30°	55′	29″
2000 :	12 <sup>h</sup>	26 <sup>m</sup>	31.0 <sup>s</sup>	+30°	38′	53″

Note: PG1224+309: “+309” is **truncated** decimal degrees without decimal (1950),  
+30°55′29″ → 30.991° → 30.9 → 309. (Standard procedure for coordinate-type names.)

### New Catalogs and Surveys:

HIPPARCOS, TYCHO-1,2 (satellite) global astrometry: positions, parallaxes, proper motions; resolved components;  $B_T$  and  $V_T$  photometry.

2MASS (2-Micron All-Sky Survey): photometric, astrometric, images; near-infrared (J,H,K<sub>s</sub> bands).

SDSS (Sloan Digitized Sky Survey): photometric, astrometric, images; optical (*ugriz* bands). Northern sky and equatorial strip, plus selected fields.

USNO-A2/USNO-B1: catalogs from digitized photographic plates: all-sky positions in J2000 (ICRF), coarse magnitudes. USNO-B1 provides proper motions.

(See *Astrophysical Quantities* 4th edition for more complete lists of surveys and catalogs.)

## Old Galactic Coordinates ( $l^I, b^I$ ).

Outmoded.

Definition: Ohlsson, 1932, Ann. Lund Obs. **3**.

North Galactic Pole:  $\text{NGP}^I = (12^h40^m, +28^\circ)$ , **1900** equinox and equator.

$l^I = 0^\circ$  at **ascending node**: no “astrophysical” meaning to  $l^I = 0^\circ$ .

## New Galactic Coordinates ( $l^{\text{II}}, b^{\text{II}}$ ).

See A.P. Lane (1979: PASP **91**, 405) for notes, history. IAU adoption in  $\approx 1958$ . Superscripts ( $^{\text{II}}$ ) no longer needed, if no possible confusion with old system.

**Exact:**  $\text{NGP}^{\text{II}}$  is  $(12^h49^m, +27.4^\circ)$ , **1950** equinox and equator.

$l^{\text{II}} = 33^\circ$  at **ascending node** as galactic equator crosses celestial equator (1950):  $(l^{\text{II}}, b^{\text{II}}) = (0^\circ, 0^\circ)$  approximates the position of the Galactic Center (as known from 21 cm radio observations).

**Approximate:**  $(l^{\text{II}}, b^{\text{II}}) = (0^\circ, 0^\circ)$  is  $(\alpha, \delta)_{1950} = (17^h42.4^m, -28^\circ55')$ . (Zombeck’s book gives this; watch out!)

Lane gives more accurately:  $(l^{\text{II}}, b^{\text{II}}) = (0, 0)$  is  $(\alpha, \delta)_{1950} = (17^h42^m, 26.603^s, -28^\circ55'0.445'')$ .

Note:  $\alpha$  differs by  $39''$ !

**Comments:** Galactic Plane $^{\text{II}}$  is tilted  $1.4866^\circ$  from Galactic Plane $^{\text{I}}$  (Lang, *Astrophysical Formulae*). The Planes cross ( $b^{\text{I}} = b^{\text{II}} = 0^\circ$ ) at  $l^{\text{I}} = 77.65^\circ, l^{\text{II}} = 109.95^\circ$ : in the Plane,  $\Delta l \approx 33^\circ$ , and  $|\Delta B| \leq 1.5^\circ$  everywhere.

## Supergalactic Coordinates.

G. de Vaucouleurs, see G. Abell in *Stars & Stellar Systems* IX, 636:

A.J. **58**, 30 (1953); *Vistas in Astronomy*, 1584 (1956); A.J. **63**, 253 (1958).

Supergalactic Pole is at  $(l^{\text{I}}, b^{\text{I}}) \equiv (15^\circ, 5^\circ)$ ,  $\rightarrow (l^{\text{II}}, b^{\text{II}}) \approx (47^\circ, 5^\circ)$ .

Supergalactic origin  $(0^\circ, 0^\circ)$  is at  $(l^{\text{I}}, b^{\text{I}}) = (105^\circ, 0^\circ)$ .

Learn where things are on the sky! Learn when they are observable! Learn when/where precession matters, and by how much! (See U&B Table 2.1, or Tirion’s Sky Atlas 2000, or a celestial globe.)

### 3. Map Projections

**3.1 References:** Standard textbooks on cartography, and J.P. Snyder & P.M. Voxland, “An Album of Map Projections” (U.S. Geological Survey Professional Paper 1453; several figures below are from this source).

**3.2 “Projections”** are not always true perspective projections, i.e., they are not always constructed by geometrical ray tracing from a point “source”. But they are always *transformations* of coordinates from the sphere to the plane. The “graticule” or grid of coordinate lines may or not map symmetrically, depending on what reference point on the sphere is chosen to be at the origin of the planar map.

**3.3 Desirable Properties:** The sphere is not “developable”, that is it cannot be flattened without stretching, tearing, or some other distortion. This is less a problem for mapping small areas of the sphere. By a suitable choice of transformation, some combination of properties can often be retained; planned uses indicate which properties to sacrifice:

- CONFORMAL: angular relationships are conserved over local regions.
- EQUAL AREA: surface areas on the map are proportional to surface areas on the sphere.
- EQUIDISTANT: distances are correct in local regions.
- UNIFORM SCALE.

**3.4 Some Azimuthal (Polar) Projections:** In the simplest case, the pole of the spherical coordinates is the origin of the planar map, parallels (lines of constant latitude) are circles and azimuths (longitude lines) are straight lines passing through the origin.

- AZIMUTHAL EQUIDISTANT: distances from the center are correct. Not a “true” projection. Often used to show one hemisphere.
- GNOMONIC: a true projection, from the center of the sphere onto a *tangent plane*. All great circles are therefore straight lines. Cannot project more than a hemisphere onto an infinite sheet. Neither conformal nor equal area. **Astronomical Use:** Standard Coordinates.
- STEREOGRAPHIC: a true projection, onto a tangent plane from the opposite point on the sphere. Conformal. All great and small circles are shown as (arcs of) circles or as straight lines. Can show entire sphere on an infinite sheet. **Mathematical Use:** the Riemann projection, used in conformal mapping of the complex plane onto a sphere. **Astronomical Use:** Planispheres, astrolabes, other uses where small shapes are desired to be shown with minimal distortion.
- ORTHOGRAPHIC: a true projection onto a plane from a point at infinity. Neither conformal nor equal area. Shows correct “perspective” view of a globe as viewed from distant space.

**3.5 Some “All Sky” Projections:** usually with a key reference point at map center, such as the direction to the Galactic Center, and the poles at the upper and lower margins of the map. The left edge of the map corresponds point by point to the right edge of the map. Very often, the most useful property of such maps is “equal area”, to allow comparison of the density of objects in different directions.

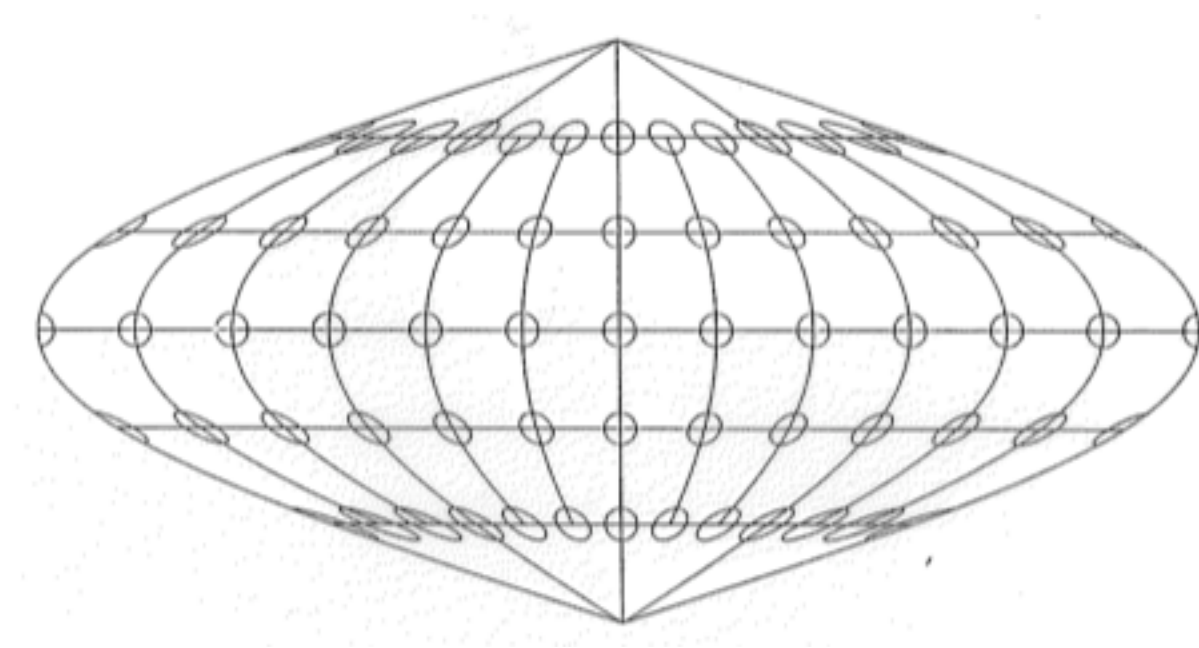
— SINUSOIDAL: simple to compute the map coordinates. “Cusps” at poles. Lines of constant “latitude” are straight and properly spaced, lines of longitude are (sine) curved. Therefore semi-useful for reading coordinates or locating objects.

— HAMMER (or, wrongly, “AITOFF”): lines of latitude and longitude are both curved, therefore less useful for reading coordinates. No cusps. Transformation equations: see, e.g., Snyder & Voxland book.

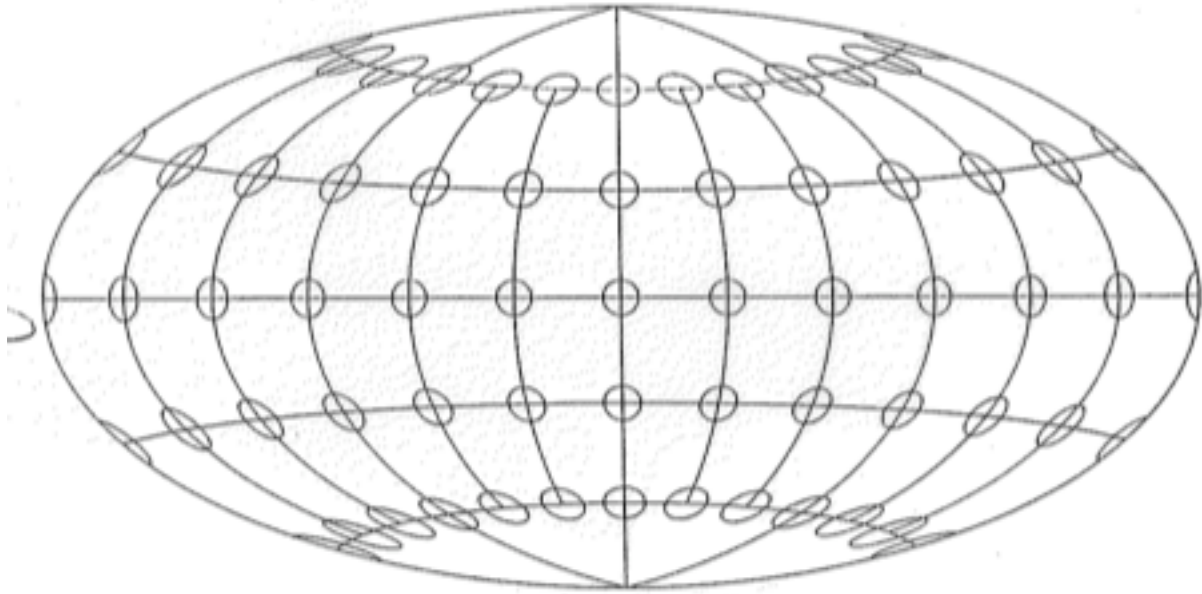
**3.6 Sky Atlases** use a variety of projections to show smallish regions of the sky, including Mercator, Transverse Mercator, various conic and polyconic projections, as well as the ones mentioned above.

#### 4. All-sky projections:

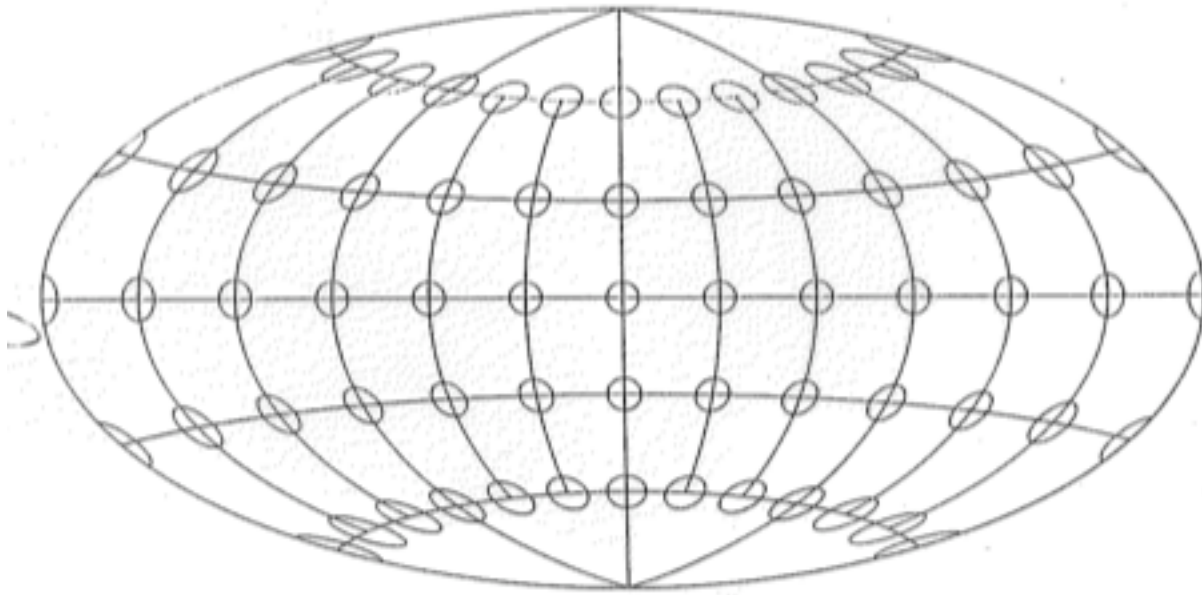
##### 4.1 Sinusoidal (equal area):



4.2 Hammer (equal area):



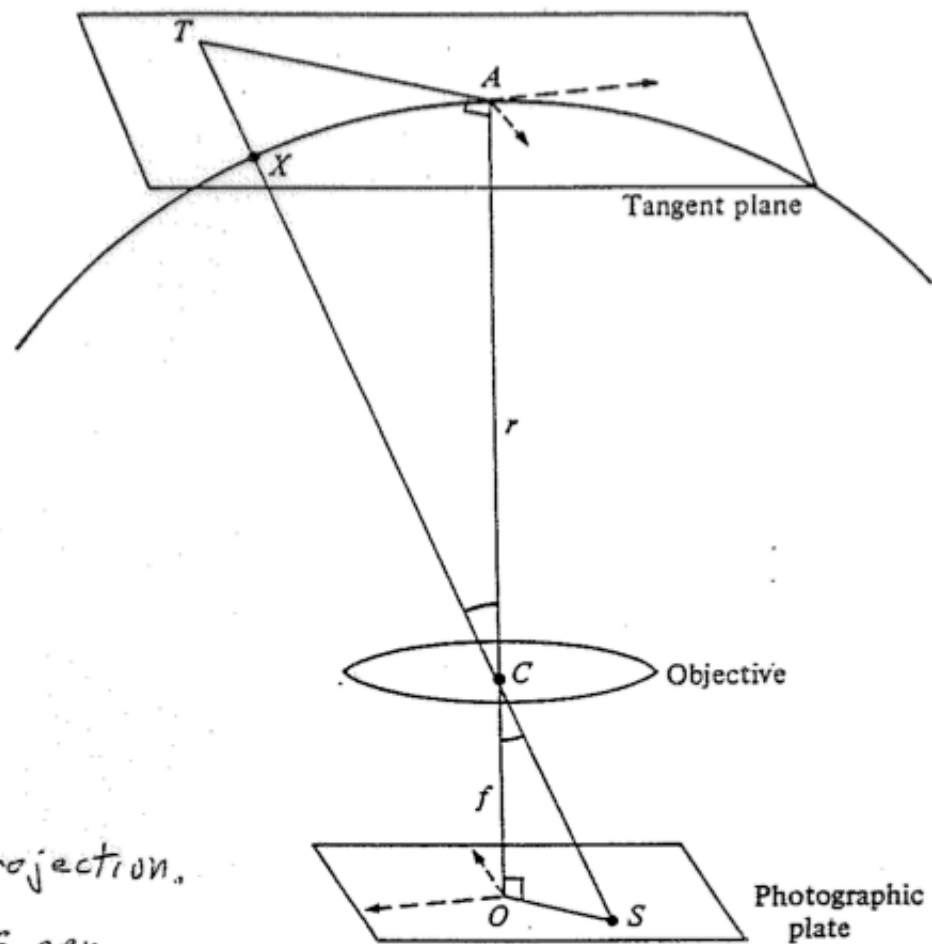
4.3 Aitoff (not equal area):





5. Gnomonic projection:

Figure 13.1 Central projection.



Gnomonic projection.  
From R.M.Green,  
Spherical Astronomy.

## 6. Standard Coordinates and Plate Constants

**References:** W.M. Smart, *Spherical Astronomy* or equivalent texts. See also H. Eichhorn 1974, *Astronomy of Star Positions*, for a thorough discussion of transforming measured plate coordinates to standard coordinates.

The problem of reducing measured positions of star images on a photographic plate or other detector to celestial coordinates is treated in two parts. First, the celestial coordinates are related to auxiliary coordinates  $\xi$  and  $\eta$ , called *standard coordinates*. Second, the standard coordinates are related to *measured coordinates*  $x$  and  $y$  by means of a transformation involving *plate constants*, which measure such things as the *plate scale*, the orthogonality of the  $x$  and  $y$  axes and their alignment with the  $\xi$  and  $\eta$  axes, centering and tilt errors, and distortions of various sorts.

**Standard coordinates.** Construct a *tangent plane* to the celestial sphere at the *tangential point*  $(\alpha_o, \delta_o)$ . According to Eichhorn (§2.3.3),

The *standard coordinates* (or *normal coordinates*)  $(\xi, \eta)$  are defined as follows: The  $\xi, \eta$  system is a rectangular Cartesian system in the plane of the plate.

The positive  $\eta$  axis is the tangent of the image of the meridian through  $(\alpha_o, \delta_o)$  at  $(\alpha_o, \delta_o)$  and points toward the image of the north celestial pole. The positive  $\xi$  axis which intersects the  $\eta$  axis in  $(\alpha_o, \delta_o)$  is a tangent to the image of the parallel through the tangential point, and points east. [In other words,  $\xi$  points E, and  $\eta$  points N, from the origin at  $(\alpha_o, \delta_o)$ .]  $\xi, \eta$  with respect to a specified  $(\alpha_o, \delta_o)$  are usually given in seconds of arc, minutes of arc, or radians. The standard coordinates in radians of an object at  $(\alpha, \delta)$  with respect to the tangential point  $(\alpha_o, \delta_o)$  are given by

$$\xi = \frac{\cos \delta \sin(\alpha - \alpha_o)}{\sin \delta_o \sin \delta + \cos \delta_o \cos \delta \cos(\alpha - \alpha_o)};$$

$$\eta = \frac{\cos \delta_o \sin \delta - \sin \delta_o \cos \delta \cos(\alpha - \alpha_o)}{\sin \delta_o \sin \delta + \cos \delta_o \cos \delta \cos(\alpha - \alpha_o)}.$$

The spherical coordinates  $(\alpha, \delta)$  of a star whose standard coordinates are  $(\xi, \eta)$  with respect to  $(\alpha_o, \delta_o)$  are obtained by solving... for  $\alpha - \alpha_o$  and  $\delta$ . One obtains

$$\cot \delta \sin(\alpha - \alpha_o) = \frac{\xi}{\sin \delta_o + \eta \cos \delta_o}$$

$$\cot \delta \cos(\alpha - \alpha_o) = \frac{\cos \delta_o - \eta \sin \delta_o}{\sin \delta_o + \eta \cos \delta_o}.$$

The derivation of these relationships is found in Smart. The  $(\xi, \eta)$  coordinates represent a gnomonic projection of  $(\alpha, \delta)$ . This projection is neither conformal nor equidistant, so that, in general, angles in the tangent plane do not equal the corresponding angles on the sphere, and neither do distances. But angles measured *at the tangential point* are equal on both plane and sphere. The scale increases away from the tangential point. Every great circle is projected to a straight line in the gnomonic projection.

**Measured coordinates.** The *focal plane* is an image by reflection of the tangent plane. Hence a relation between measured coordinates  $(x, y)$ , which might be micrometer measures or pixel coordinates on a CCD image, and  $(\xi, \eta)$  is usually modeled as a linear transformation of the type

$$\begin{aligned}\hat{\xi} &= Ax + By + C, \\ \hat{\eta} &= -Bx + Ay + D\end{aligned}$$

if  $x$  and  $y$  are known to be orthogonal and to have the same scale (“Four-constant Model”). An equivalent transformation is

$$\begin{aligned}\hat{\xi} &= \rho(x \cos \sigma + y \sin \sigma) + C, \\ \hat{\eta} &= \rho(-x \sin \sigma + y \cos \sigma) + D,\end{aligned}$$

where  $\rho$  is the *scale factor* and  $\sigma$  is the *orientation angle*.

More generally,

$$\begin{aligned}\hat{\xi} &= Ax + By + C, \\ \hat{\eta} &= Dx + Ey + F\end{aligned}$$

when the axes are allowed to be skew and not of equal scale (“Six-constant Model”). Still more general models are possible, e.g. involving higher order terms that reflect a scale change with distance from plate center, as is necessary for wide-field Schmidt plates.

The problem of ascertaining the *plate constants*  $A, B, C, \dots$  involves *astrometric standard stars* whose  $(\alpha, \delta)$  and hence  $(\xi, \eta)$  coordinates are known, and whose  $(x, y)$  positions have been measured. A least-squares adjustment of the plate constants to minimize errors in  $(\hat{\xi}, \hat{\eta})_{std}$  is carried out. The standard coordinates of *unknown* stars can then be estimated from the measured  $(x, y)$  positions, and  $(\alpha, \delta)$  can then be found, for the *epoch* of the plate. Note that  $\alpha_o$  and  $\delta_o$  are actually part of the model, but are improved iteratively, separate from finding the plate constants.

Note also that  $(\alpha, \delta)$  and  $(\xi, \eta)$  can be chosen to refer to any appropriate *equinox*, but if the “plate” is aligned N–S at the telescope, then the *equinox of date* is an appropriate choice. To reduce the inferred  $(\alpha, \delta)$  of date to a standard equinox involves computing precession.